Finite-Q^2 corrections in PVDIS

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3rd International Workshop on Nucleon Structure at Large Bjorken x

Newport News, 10/13/2010

A Road Map

Why are small Q^2 corrections relevant at high x?

An inventory of corrections to the PVDIS signal at and beyond LO in α_s

The particular challenge posed by the target mass

Proceeding on the experimental front

The interest in large x_B physics

An array of high-x observables potentially discriminate various quark models --- e.g., the PDF ratio ~ d/u

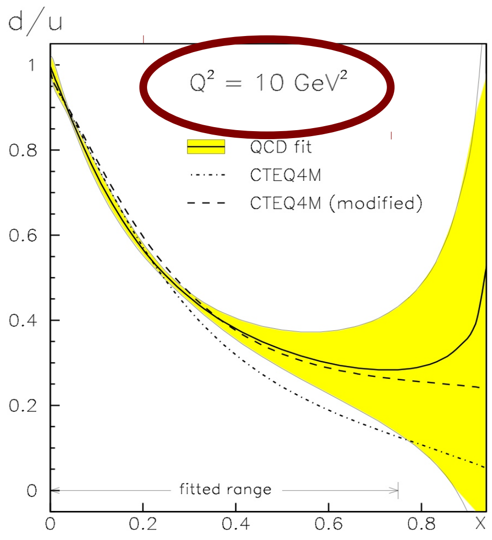
Sizable uncertainties remain, particularly at threshold, $x_B \to 1$

e.g., scalar di-quark model (in which total spin zero behavior dominates)

$d(x) / u(x) \to 0$ for $x_B \to 1$

ALSO: QCD evolution – physics at low-Q^2, high-x filters to higher Q^2, lower x_B

Intermediate



PVDIS at finite Q^2

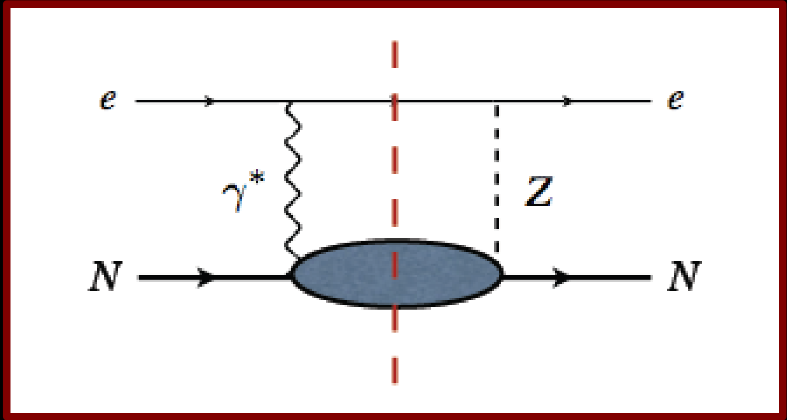
T.H. and W. Melnitchouk, Phys. Rev. D **77**, 114023 (2008)

PVDIS in the SU(2) U(1) theory

$$\mathcal{L}^{\rm PV} = \frac{G_F}{\sqrt{2}}[\bar{e}\gamma^\mu\gamma_5 e(C_{1u}\bar{u}\gamma_\mu u + C_{1d}\bar{d}\gamma_\mu d) + \bar{e}\gamma^\mu e(C_{2u}\bar{u}\gamma_\mu\gamma_5 u + C_{2d}\bar{d}\gamma_\mu\gamma_5 d)]$$

Interesting physics resides principally in electroweak couplings

$$C_{1u} = g_A^e \cdot g_V^u = -\tfrac{1}{2} + \tfrac{4}{3}\sin^2\theta_W,$$
$$C_{1d} = g_A^e \cdot g_V^d = \tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W,$$
$$C_{2u} = g_V^e \cdot g_A^u = -\tfrac{1}{2} + 2\sin^2\theta_W,$$
$$C_{2d} = g_V^e \cdot g_A^d = \tfrac{1}{2} - 2\sin^2\theta_W.$$



γ-Z interference allows study of flavor dependence in the nucleon

The parity-violating asymmetry is a natural PVDIS observable

$$A^{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

Expanding in terms of EW structure functions

$$A^{PV} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\frac{g_A^e(2xyF_1^{\gamma Z} - 2[1 - 1/y + xM/E]F_2^{\gamma Z}) + g_V^e x(2-y)F_3^{\gamma Z}}{2xyF_1^{\gamma} - 2[1 - 1/y + xM/E]F_2^{\gamma}}$$

Note: A^{PV} ~ interference / electromagnetic

Callan-Gross breaking encoded in parameter R

$$F_2^{(0)} = 2xF_1^{(0)}$$

⇒

unknown phenomenology

$$R^{\gamma(\gamma Z)} \equiv \frac{\sigma_L^{\gamma(\gamma Z)}}{\sigma_T^{\gamma(\gamma Z)}} = r^2\,\frac{F_2^{\gamma(\gamma Z)}}{2xF_1^{\gamma(\gamma Z)}} - 1$$

At SF-level the PV asymmetry becomes

$$A^{\rm PV} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)\left[g_A^e Y_1 \frac{F_1^{\gamma Z}}{F_1^\gamma} + \frac{g_V^e}{2} Y_3 \frac{F_3^{\gamma Z}}{F_1^\gamma}\right]$$

$$Y_1 = \frac{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma Z})) - 2xyM/E}{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right)$$

$$Y_3 = \frac{1-(1-y)^2}{1+(1-y)^2 - y^2(1-r^2/(1+R^{\gamma})) - 2xyM/E}\left(\frac{r^2}{1+R^{\gamma}}\right) .$$

more generally ---

$$A^{\rm PV} = -\left(\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\right)(Y_1 a_1 + Y_3 a_3)$$

dependence on flavor/current couplings resides in $a_{1,\,3}$

target-specific values for $a_{1,3}$ yield:

PROTON

$$a_1^p = \frac{12C_{1u} - 6C_{1d}d/u}{4 + d/u}$$

$$a_3^p = \frac{12C_{2u} - 6C_{2d}d/u}{4 + d/u}$$

iso-scalarity of the deuteron target ⇒

DEUTERON

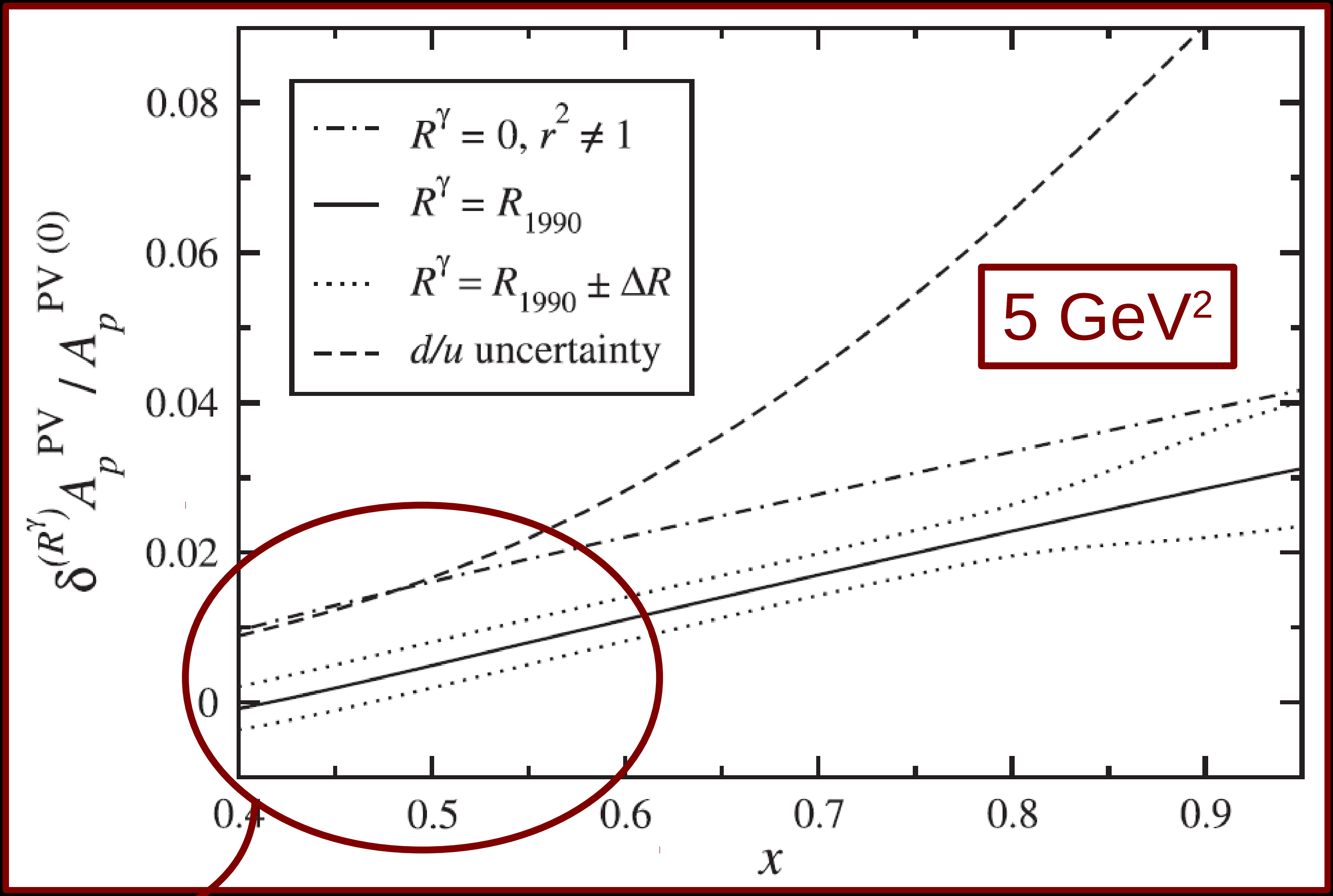
$$a_1^d = \frac{6}{5}(2C_{1u} - C_{1d})$$
$$a_3^d = \frac{6}{5}(2C_{2u} - C_{2d})$$

$$A^{PV} = -\left(\frac{3G_F Q^2}{10\sqrt{2}\pi\alpha}\right)\left[Y_1(2C_{1u} - C_{1d}) + Y_3(2C_{2u} - C_{2d})\right]$$

only source of flavor dependence; precise determination of R parameters needed

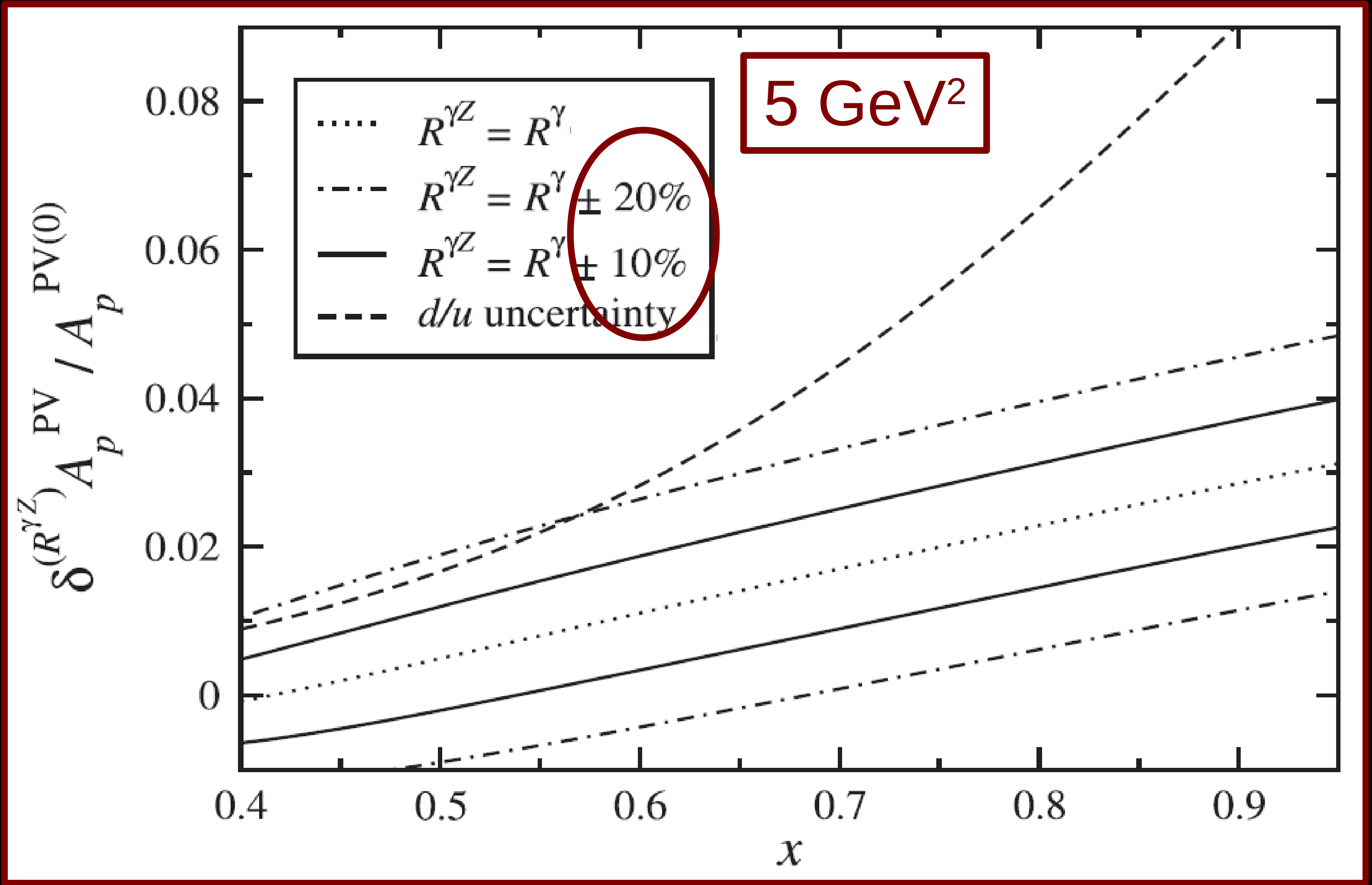
As a first pass, sensible approach is to investigate sensitivity of A^{PV} to hypothetical R parameter behaviors

R^{γ} sensitivity



R parameter uncertainty competes with d/u signal at low x_B

$R^{\gamma Z}$ has an undetermined phenomenology; how might it behave?



Conservative error bands motivated by standing phenomenology of R^{γ} and R^{Z}

S. A. Kulagin and R. Petti, Phys. Rev. D **76**, 094023 (2007).

What physics might break $R^{\gamma Z} \equiv R^{\gamma}$?

One-loop and higher-order perturbative corrections

Preliminary calculations done here

Kinematic higher-twist corrections; i.e., target mass effects

⇒ Other forms of *non*-perturbative physics

Target Mass Corrections

A. Accardi, W. Melnitchouk, and TH; *in progress*.

The OPE approach of Georgi-Politzer

$$\int d^4x \; e^{iq\cdot x} \langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^{\nu}q^{\mu_2} + q^{\mu}q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}} \frac{2^{2k}}{Q^{4k}} A_{2k} \Pi_{\mu_1\cdots\mu_{2k}}$$

$$\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle$$

local operators

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j} \text{ terms})$$

$$= \sum_{j=0}^{k}(-1)^j \frac{(2k-j)!}{2^j(2k)^j} g\cdots g \; p\cdots p$$

traceless, symmetric rank-$2k$ tensor

O. Nachtmann, Nucl. Phys. B **63**, 237 (1973)

$$M_2^n(Q^2) = \int dx\ x^{n-2}\ F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

Mass-dependent structure function extracted from expansion term via Mellin transform

$$F_1^{\rm OPE}(x, Q^2) = \frac{x}{\xi\rho} F_1^{(0)}(\xi, Q^2) + \frac{M^2 x^2}{Q^2 \rho^2} \int_\xi^1 du \frac{F_2^{(0)}(u, Q^2)}{u^2} + \frac{2M^4 x^3}{Q^4 \rho^3} \int_\xi^1 dv (v - \xi) \frac{F_2^{(0)}(v, Q^2)}{v^2},$$

$$F_2^{\rm OPE}(x, Q^2) = \frac{x^2}{\xi^2 \rho^3} F_2^{(0)}(\xi, Q^2) + \frac{6M^2 x^3}{Q^2 \rho^4} \int_\xi^1 du \frac{F_2^{(0)}(u, Q^2)}{u^2} + \frac{12 M^4 x^4}{Q^4 \rho^5} \int_\xi^1 dv (v - \xi) \frac{F_2^{(0)}(v, Q^2)}{v^2},$$

$$F_3^{\rm OPE}(x, Q^2) = \frac{x}{\xi \rho^2} F_3^{(0)}(\xi, Q^2) + \frac{2M^2 x^2}{Q^2 \rho^3} \int_\xi^1 du \frac{F_3^{(0)}(u, Q^2)}{u},$$

Introduce modified scaling

$$\xi(x, Q^2) = \frac{2x}{1+\rho}, \qquad \rho = \sqrt{1 + 4M^2 x^2 / Q^2}$$

Problems at the $x_B \to 1$ kinematic threshold

… the OPE prescription is not without flaws ...

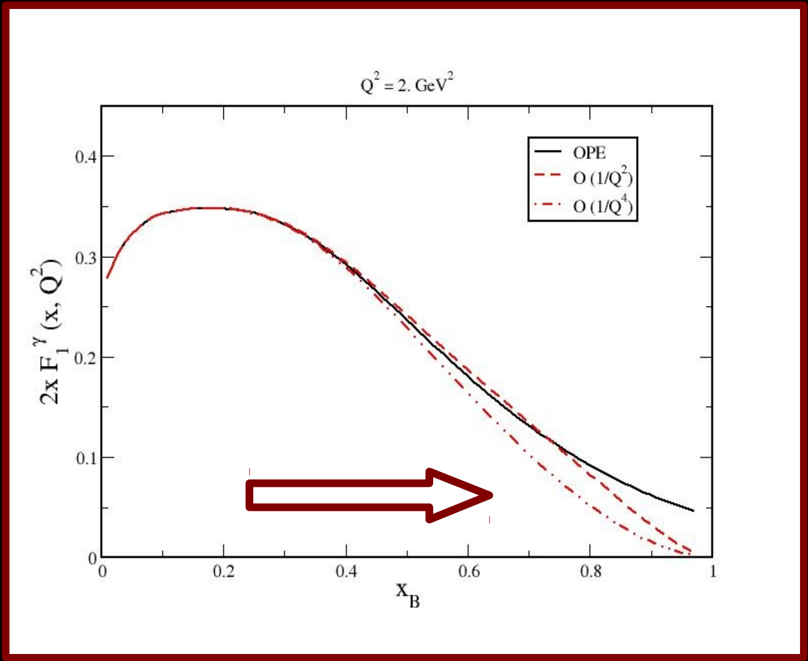
in particular, rescaling introduces some non-physical behavior at *threshold*:

$$\xi = \frac{2x_B}{1+\sqrt{1+4x_B^2 M^2/Q^2}}$$

$$\xi_0 \equiv \xi(x=1) < 1$$

implies ...

$$F_i^{\mathrm{TMC}}(x=1, Q^2) > 0$$



How to solve the problem?

Contributions from HT proposed to make up the ground

The Collinear Factorization Formalism

$$p^\mu = p^+ \bar{n}^\mu + \frac{M^2}{2p^+} n^\mu$$

$$q^\mu = -\xi p^+ \bar{n}^\mu + \frac{Q^2}{2\xi p^+} n^\mu$$

$$k^\mu = x p^+ \bar{n}^\mu + \frac{m_f^2}{2xp^+} n^\mu$$

decompose the DIS handbag diagram in light-cone coord.

Natural choice: set (p, q) collinear

Factorized helicity SFs

$$F_i(x, Q^2) = \sum_f \int_{x_{min}}^{x_{max}} \frac{dx}{x} h_i^f(x_f, Q^2) \phi_{f/N}(x, Q^2)$$

$$F_1(x) = h_1^f \bigotimes \phi_{f/N}(\xi)$$

Corrections to SFs independent of exchange boson

$$F_3(x) = \frac{\rho_f}{\rho_B} h_3^f \bigotimes \phi_{f/N}(\xi)$$

$$F_2(x) = \frac{x_B}{x_f} \frac{\rho_f^2}{\rho_B^2} h_2^f \bigotimes \phi_{f/N}(\xi)$$

A. Accardi and J. W. Qiu, **JHEP 0807** (2008) 090.

An alternative: expansion of the LT OPE

Expand to $O(1/Q^2)$ to "brute force" high-x SFs to observe proper threshold behavior:

$$F_1^{\rm OPE}(x,Q^2) \approx F_1^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(2\int_x^1 \frac{dz}{z^2}F_1^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_1^{(0)}(x,Q^2)\right)$$

$$F_2^{\rm OPE}(x,Q^2) \approx \left(1-\frac{4x^2M^2}{Q^2}\right)F_2^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(6\int_x^1 \frac{dz}{z^2}F_2^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_2^{(0)}(x,Q^2)\right) ,$$

$$F_3^{\rm OPE}(x,Q^2) \approx \left(1-\frac{2x^2M^2}{Q^2}\right)F_3^{(0)}(x,Q^2) + \frac{x^3M^2}{Q^2}\left(2\int_x^1 \frac{dz}{z^2}F_3^{(0)}(z,Q^2) - \frac{\partial}{\partial x}F_3^{(0)}(x,Q^2)\right) .$$

TWIST: dimension *minus* spin

⇒ Expansion in $1/Q^2$ simulates inclusion of HT contributions

$$F_1^{O(1/Q^4)}(x,Q^2) = \ldots \frac{M^4}{Q^4}(2x^3 g_2(x,Q^2) - 4x^4 h_2(x,Q^2) + x^3 F_2^{(0)}(x,Q^2) + 3x^4 F_1^{(0)}(x,Q^2) + 3x^5 F_1^{'(0)}(x,Q^2) + \frac{x^6}{2} F_1^{''(0)}(x,Q^2))$$

Expand the OPE result to next order in $1/Q^2$

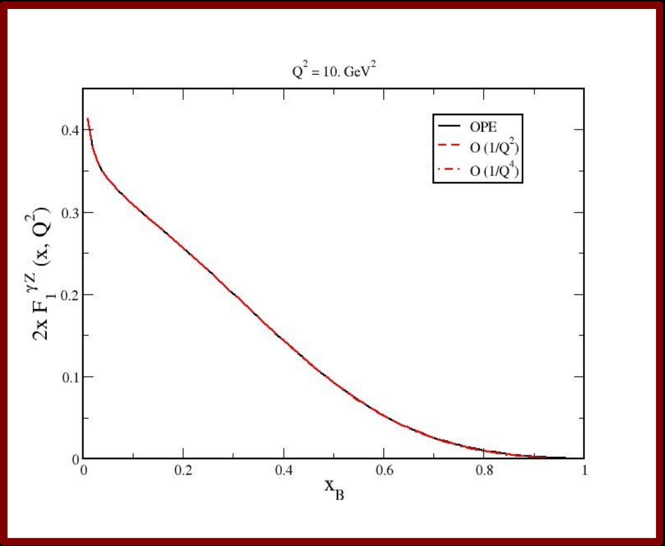
$$F_2^{O(1/Q^4)}(x,Q^2) = \ldots \frac{M^4}{Q^4}(12x^4 g_2(x,Q^2) - 48x^5 h_2(x,Q^2) + 23x^4 F_2^{(0)}(x,Q^2) + 6x^5 F_2^{'(0)}(x,Q^2) + \frac{x^6}{2} F_2^{''(0)}(x,Q^2))$$

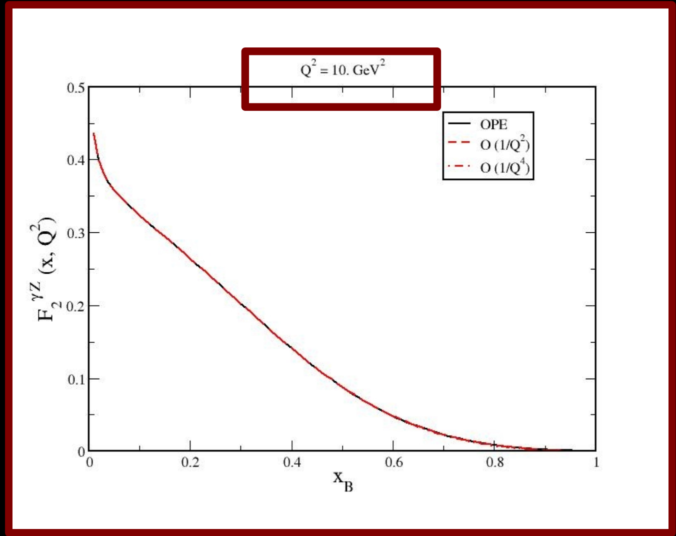
to what extent does the qualitative description depend on the $O^{(n)}(1/Q^2)$?

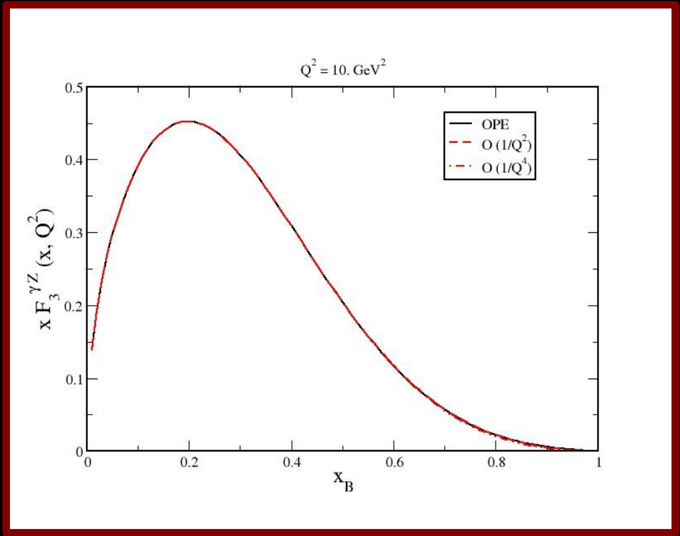
$$F_3^{O(1/Q^4)}(x,Q^2) = \ldots \frac{M^4}{Q^4}(-12x^4 h_3(x,Q^2) + 13x^4 F_3^{(0)}(x,Q^2) + 5x^5 F_3^{'(0)}(x,Q^2) + \frac{x^6}{2} F_3^{''(0)}(x,Q^2))$$

At intermediate $Q^2 \sim 10$ GeV^2, $1/Q^2$ expansions are well-behaved and convergent to OPE-generated corrections

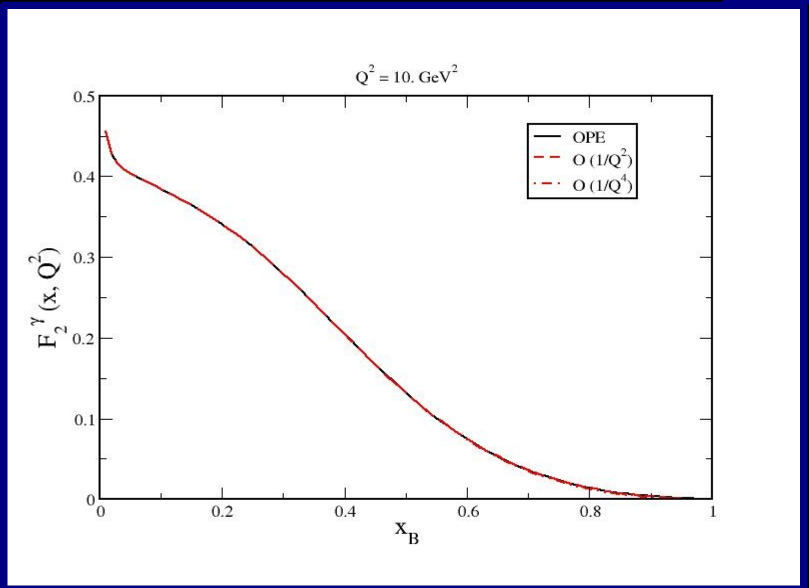
Electroweak SFs



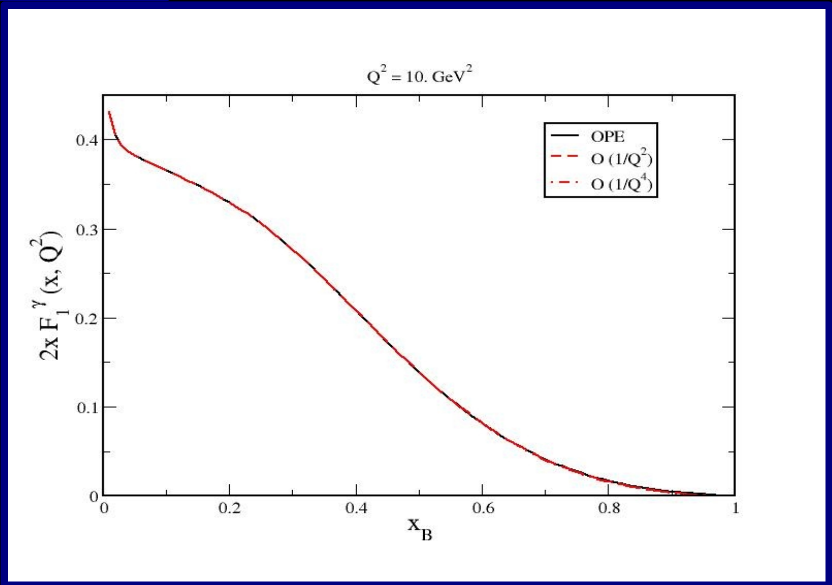


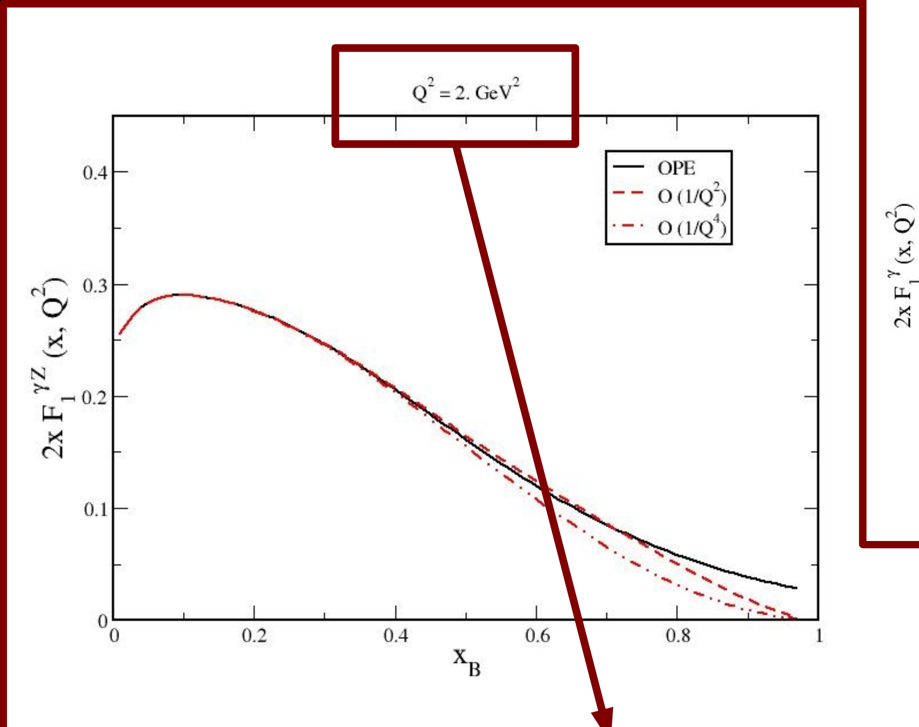


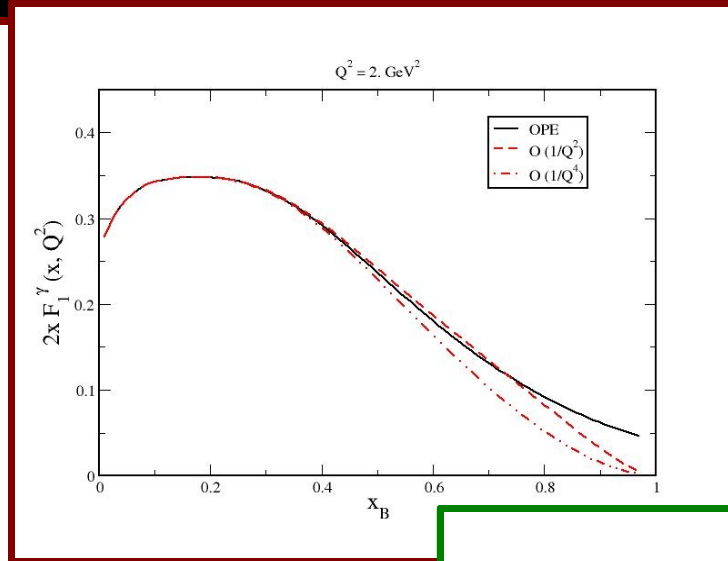
Electromagnetic SFs



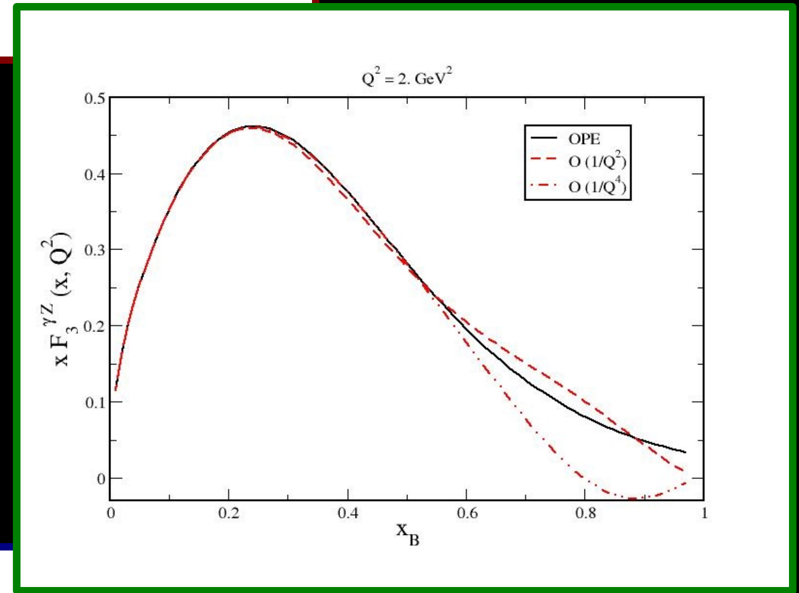
10 GeV^2



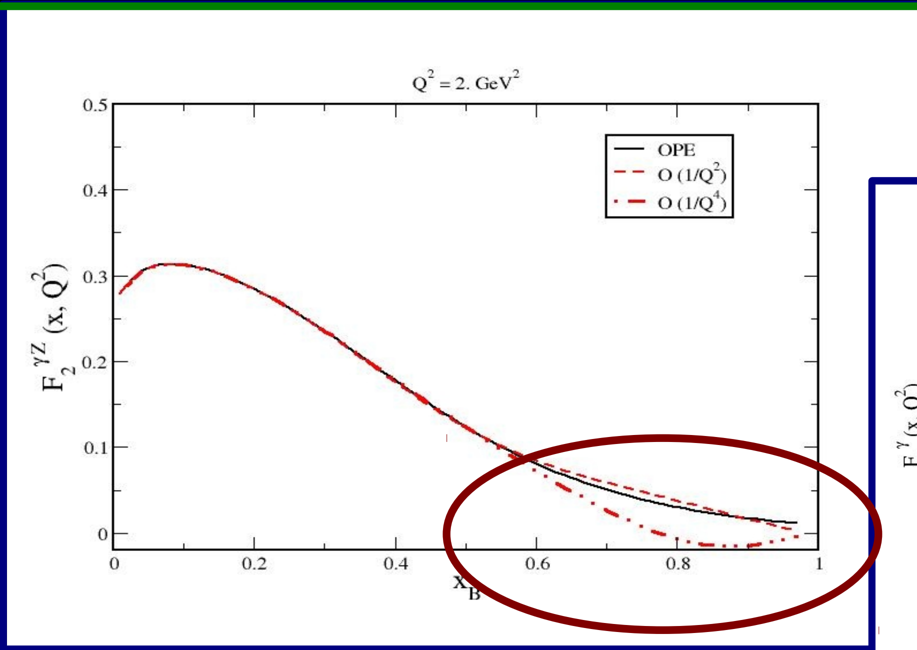


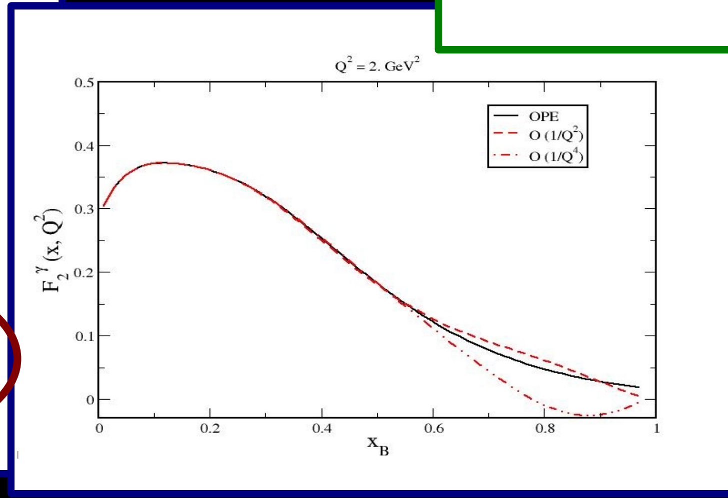


2 GeV^2



At low Q^2, high-x_B behavior become distinct



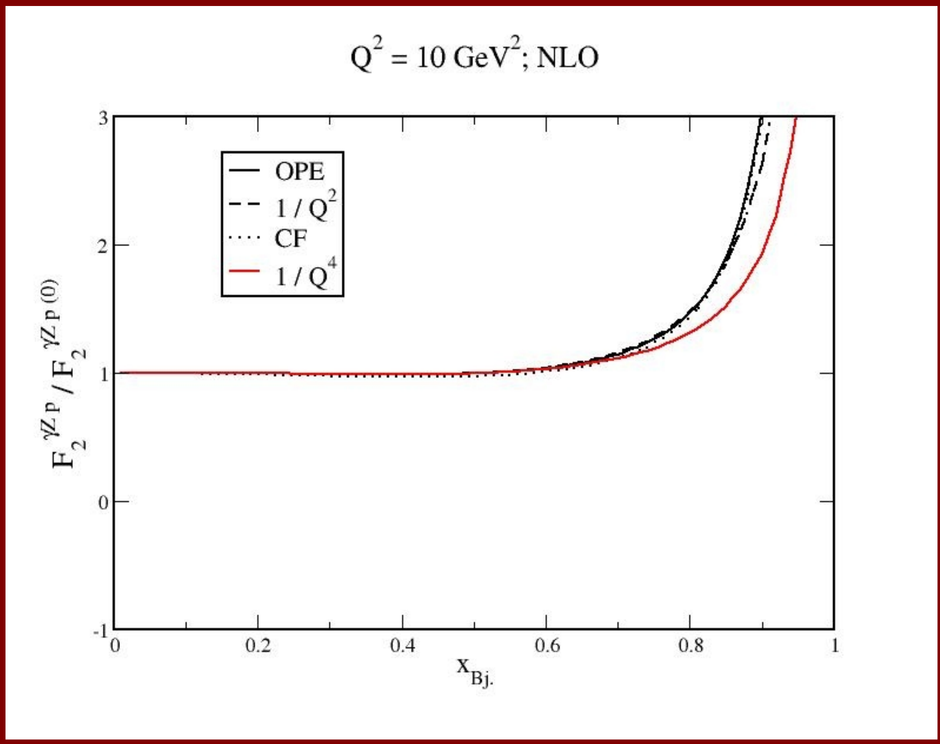


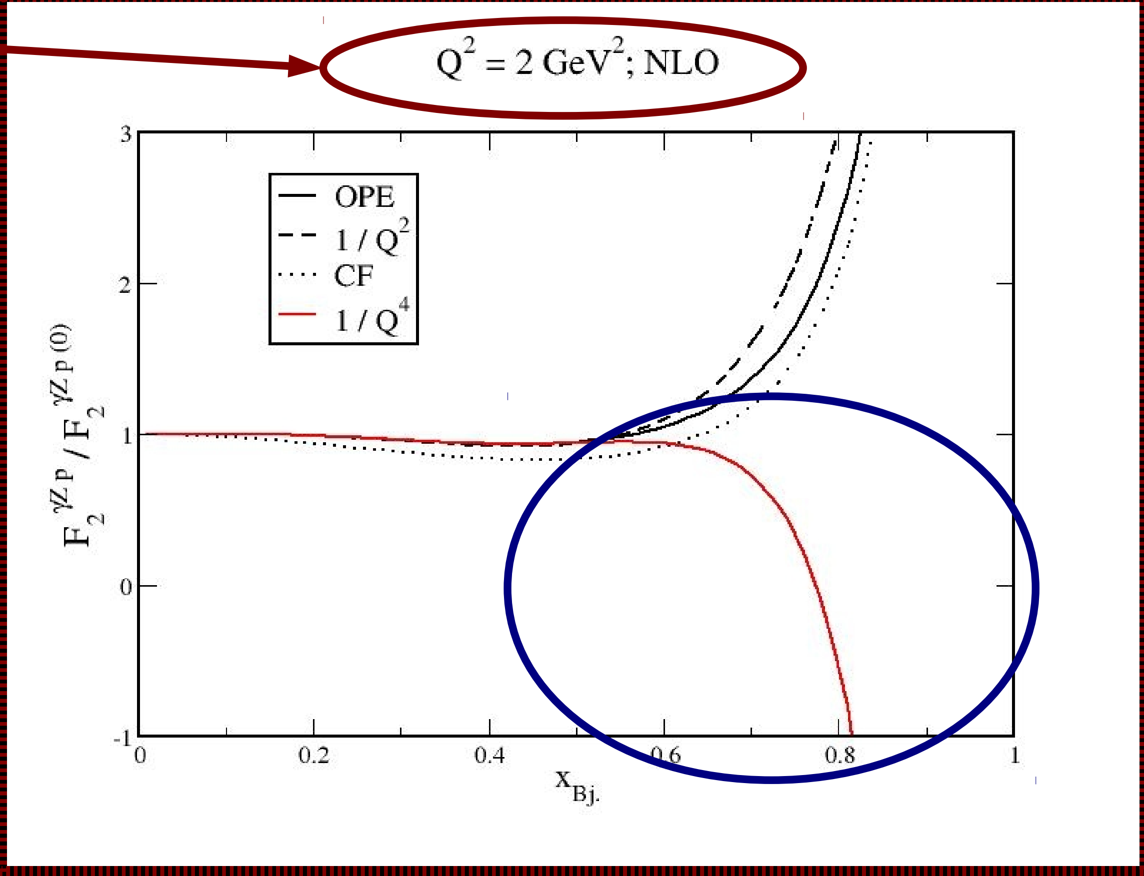
1/Q^2 expansion of OPE yields unphysical threshold behavior when computed at higher order

In general, one expects structure functions to be positive definite

Move to lower Q^2

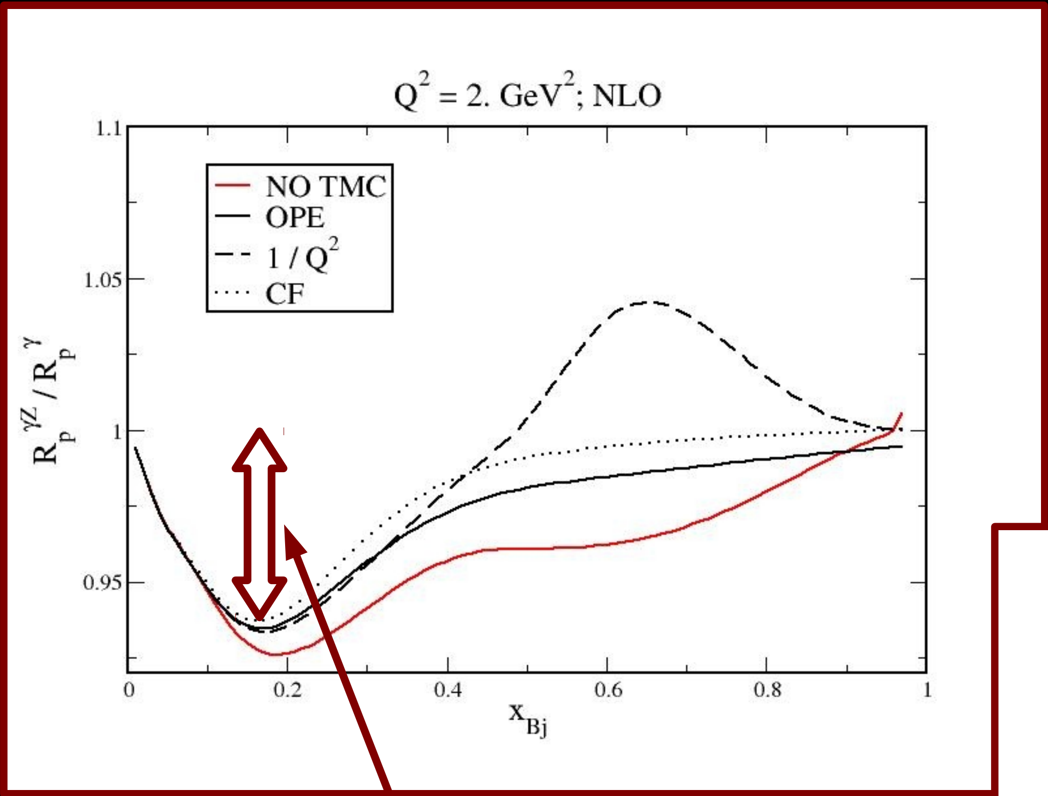
Expansion parameter becomes sizable





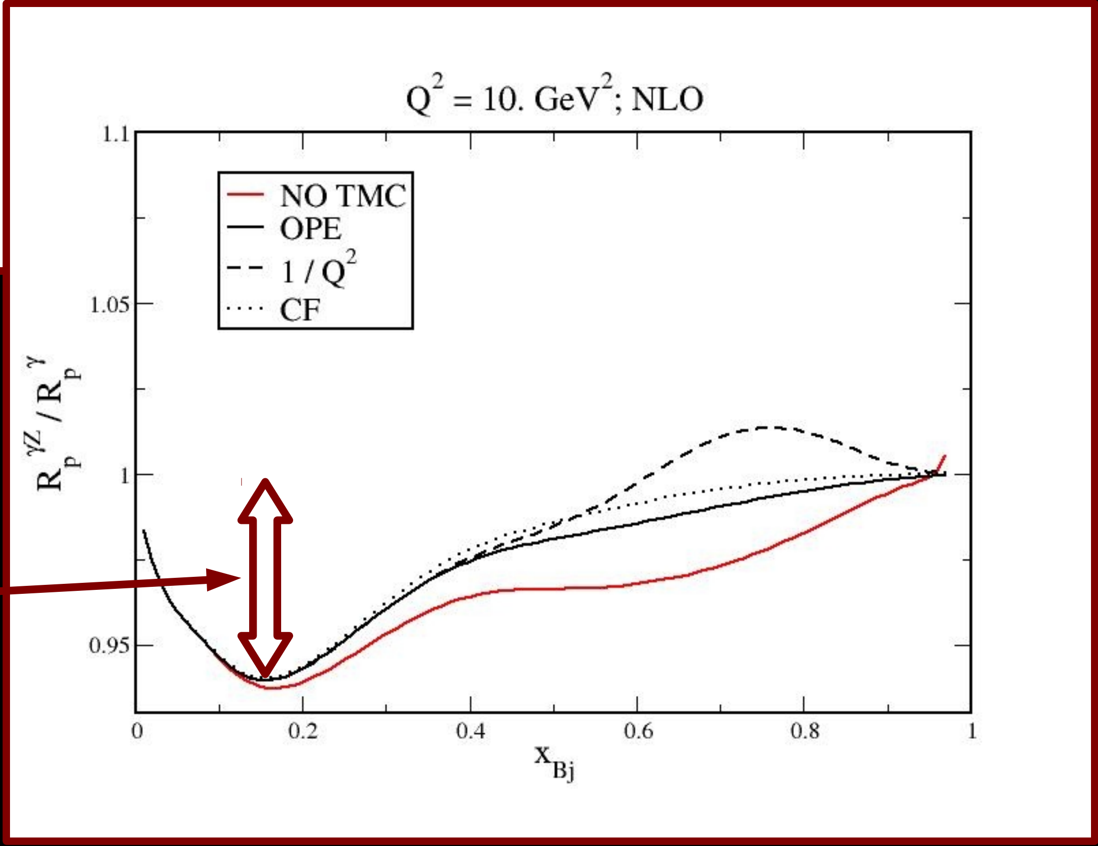
Mass effects in physical observables

What breaks the Callan-Gross relation?



At LO in the massless limit parton model, Callan-Gross is exact

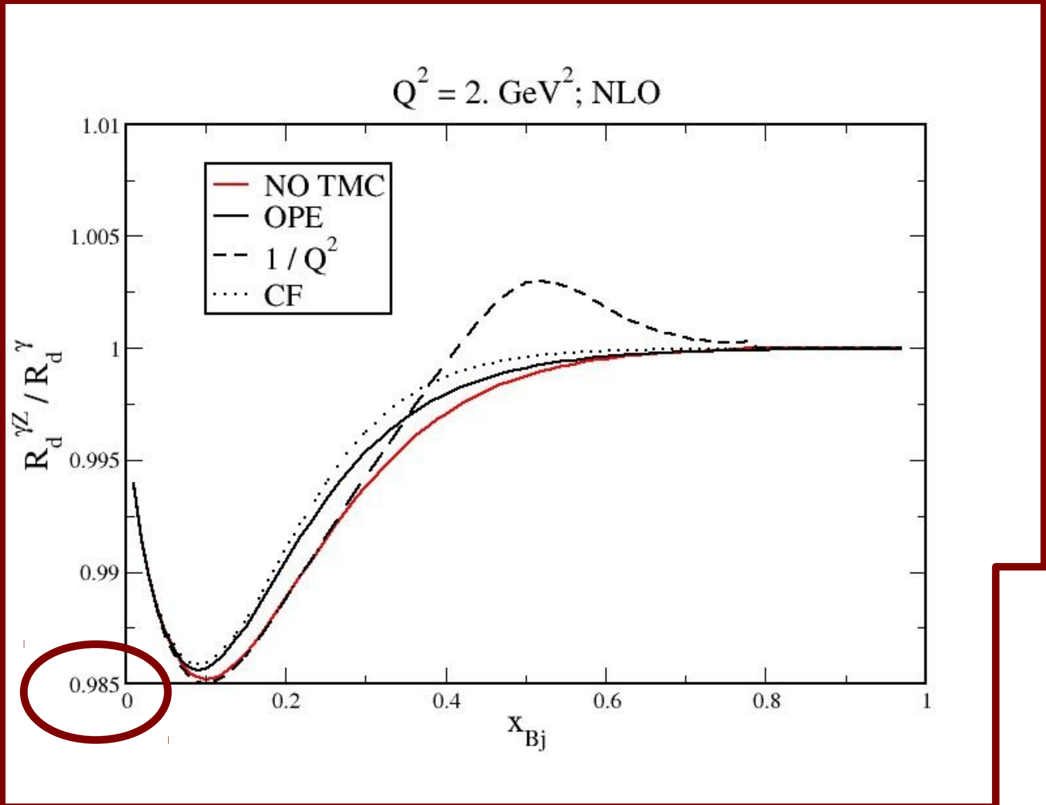
$$F_2^{(0)} = 2x F_1^{(0)}$$



Perturbative corrections in α_s and TMCs generate a ~ 5% effect

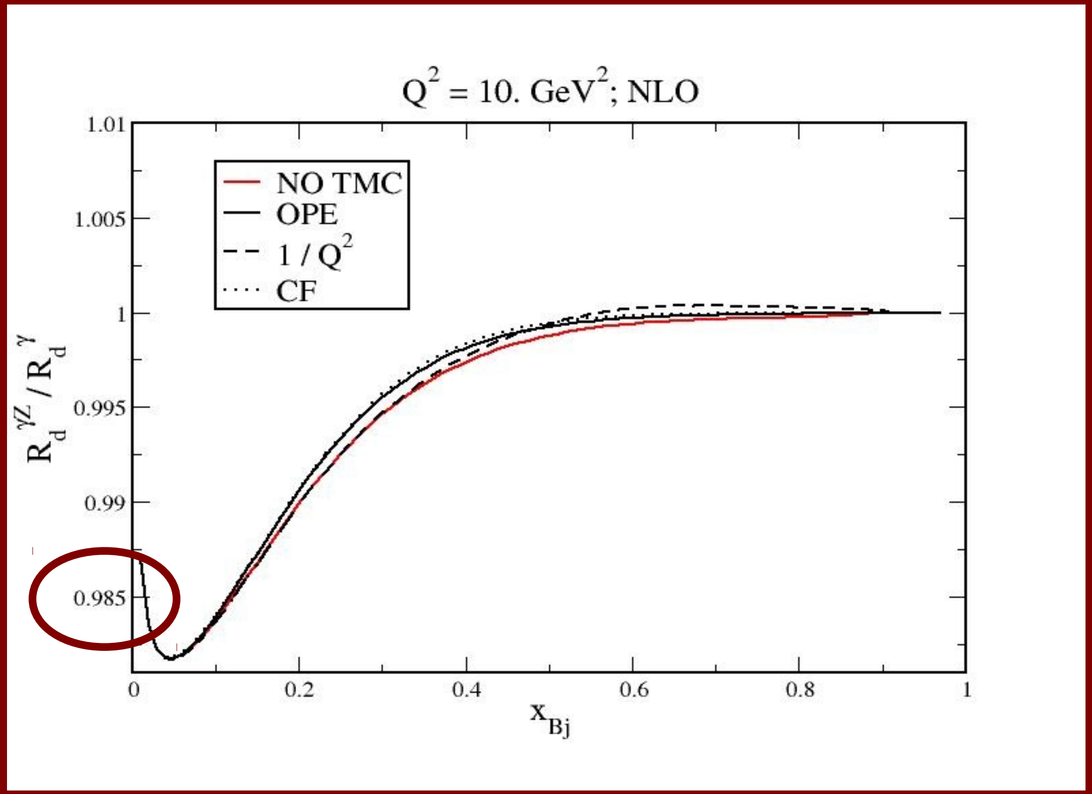
PROTON

Isoscalarity of the deuteron diminishes this effect



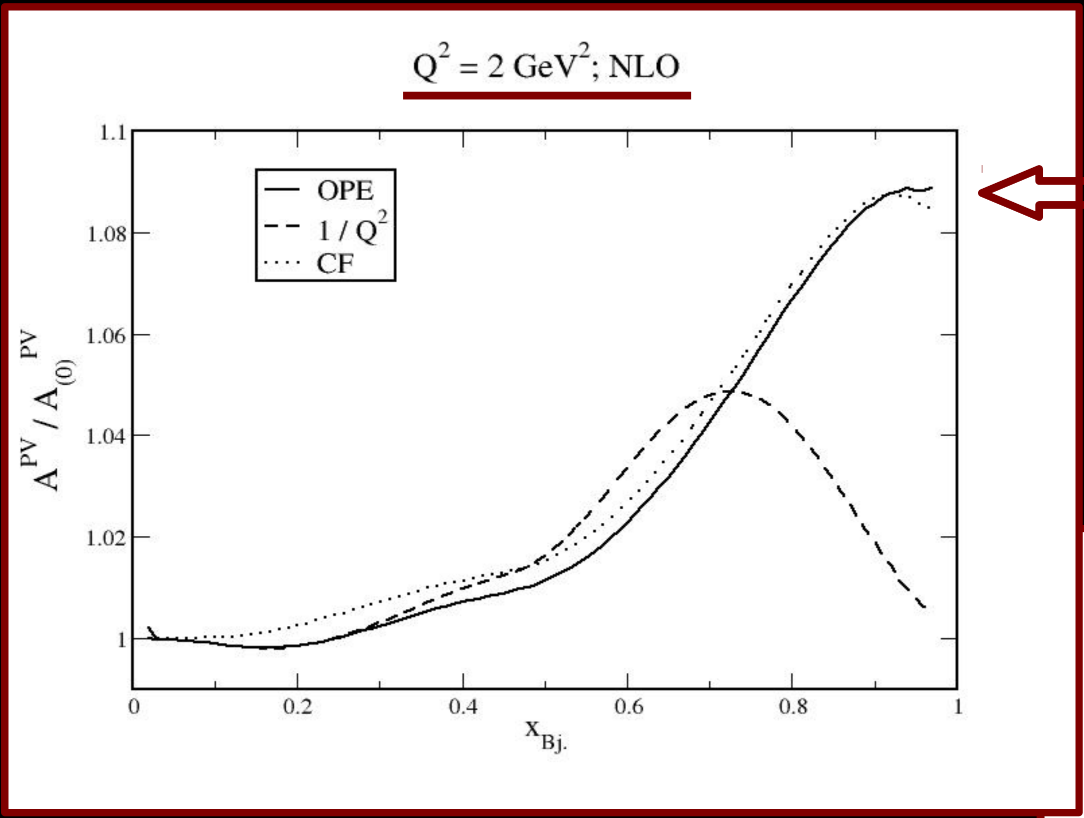
Like the proton calculation $R^{\gamma Z} = R^{\gamma}$ is broken modestly in general

largest effect at low x_B

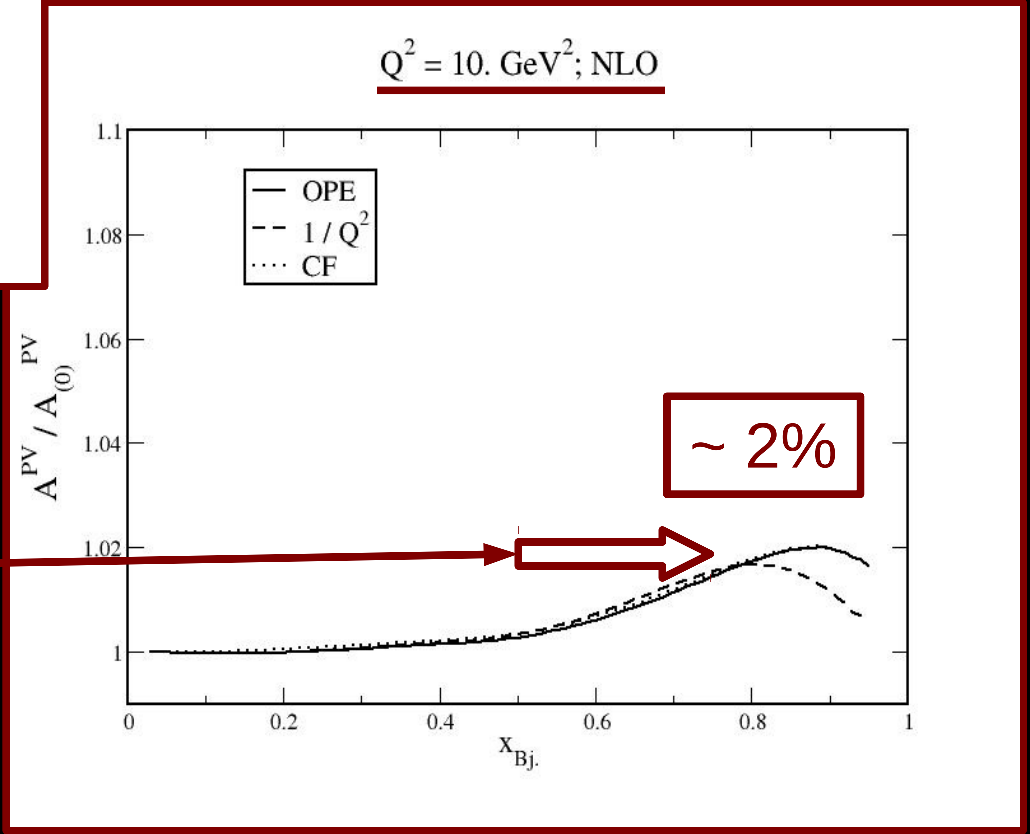


Cancellation of flavor - dependent effects → $R^{\gamma Z} \approx R^{\gamma}$ within ~ 2%

Mass corrections in proton target A^{PV}

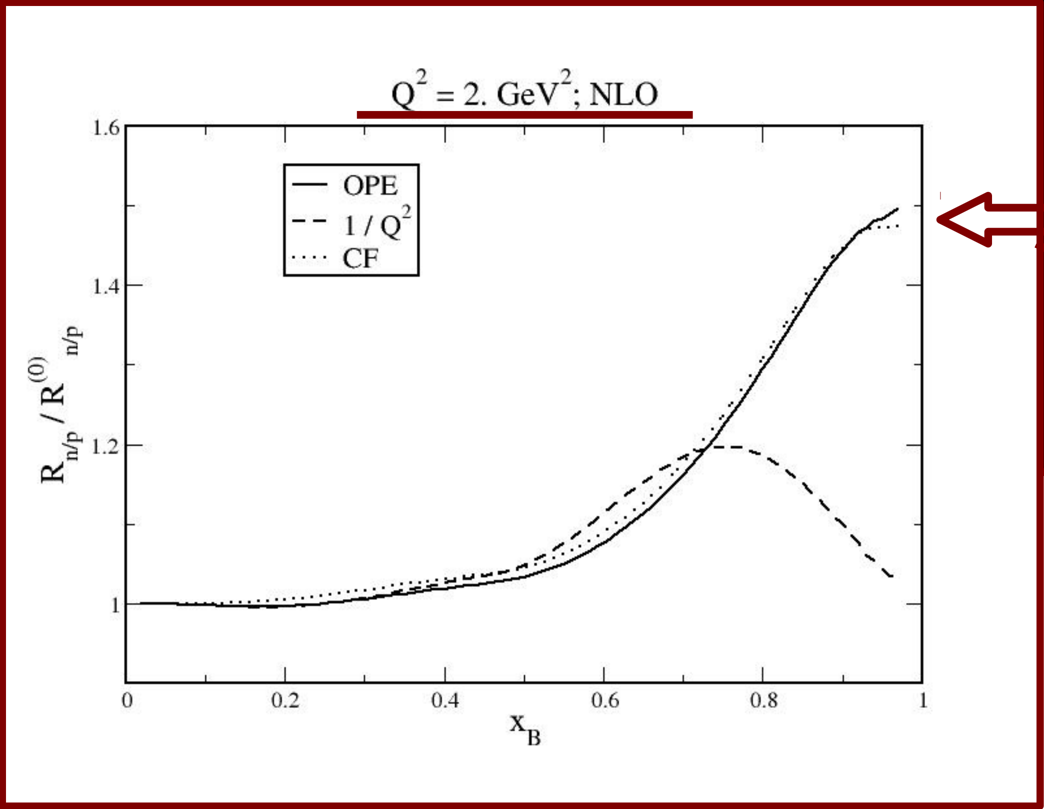


Target mass effect enhanced by ~ 8% in the limit $x_B \rightarrow 1$



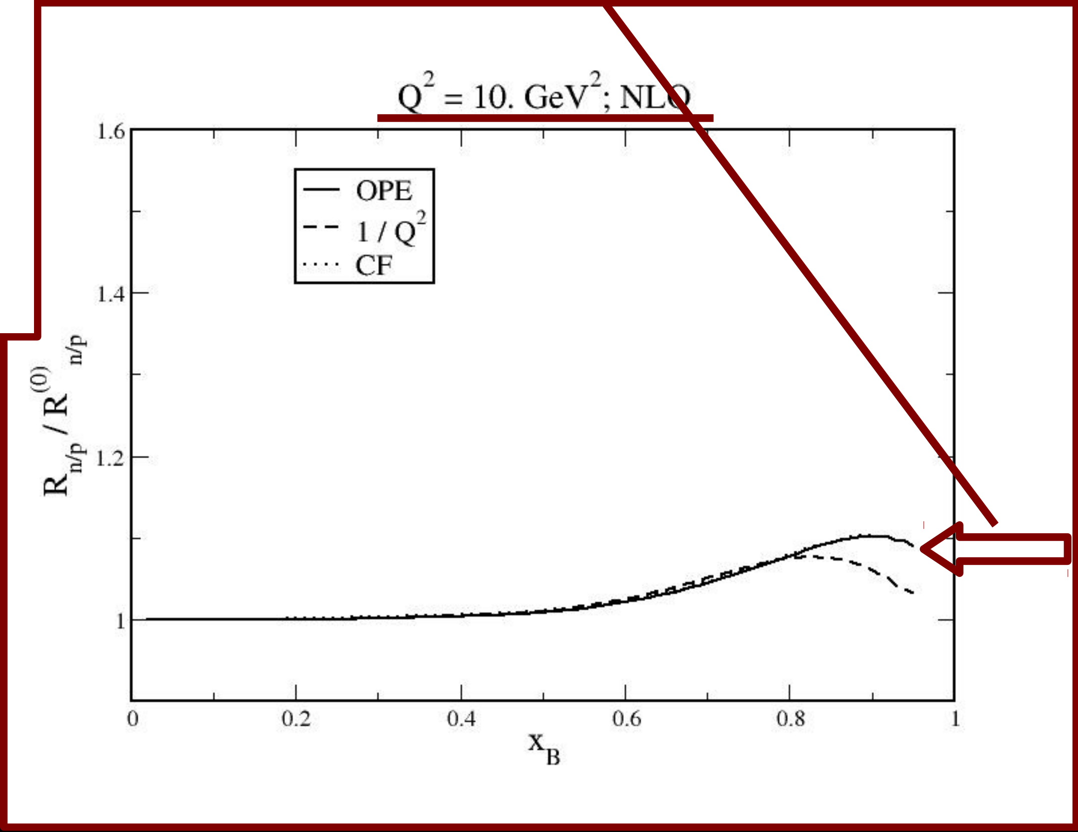
mass effect shrinks rapidly at Q^2 ~ 10 GeV^2; less prescription dependence in TMCs

Aside: phenomenology of $R_{n/p}$



$$\frac{F_2^{n\gamma Z}}{F_2^p} = \frac{C_{1d}e_d + C_{1u}e_u(\frac{q_d}{q_u})}{C_{1u}e_u + C_{1d}e_d(\frac{q_d}{q_u})}$$

FACTOR OF FIVE



note the qualitative similarity to the A^{PV} mass effect

What about A_d^{PV} ?

Large-scale flavor cancellation leads to a ~ zero target mass effect in deuteron

$$A^{PV} = -\left(\frac{3G_F Q^2}{10\sqrt{2}\pi\alpha}\right)[Y_1(2C_{1u} - C_{1d}) + Y_3(2C_{2u} - C_{2d})]$$

DEUTERON IS ISOSCALAR

Unlike the proton, $A_d^{(TMC)} / A_d^{(0)} \sim 1$ over a range of x_B and Q^2

Good new for experimental efforts ...

Concluding thoughts

For deuteron targets especially, TMCs and NLO physics are unlikely to generate violations of Callan-Gross sufficient to endanger experiments

Still, other sources of Callan-Gross-violating physics remain largely unexplored

When in doubt, move to high Q^2 to avoid these complications

THE END